For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Let's start off with a simple one! Find the sum of all the values in the five number summary of: 85, 43, 63, 4, 73, 87, 4, 22, 43, 646, 2, 32.

   (A) 839 (B) 674 (C) 783 (D) 754 (E) NOTA

2. The pdf of random variable X is defined by the equation $P(X = x) = \sqrt{r^2 - (x - 10)^2}$. Find the value of r that makes this pdf valid. Round to three decimal places.

   (A) 0.321 (B) 0.721 (C) 0.798 (D) 0.815 (E) NOTA

3. Prabhas is making the long journey from his home, at (0, 0), to school, at (6, 6). Because of the way the roads are laid out, if he is at the point (x, y) he can only ever move to the point $(x + 1, y)$ or $(x, y + 1)$. Prabhas chooses a path randomly from all possible paths that would get him to school. Find the probability that Prabhas's path does not touch or cross the freeway, represented by the line $y = x - 2$. Round to three decimal places.

   (A) 0.464 (B) 0.489 (C) 0.512 (D) 0.559 (E) NOTA

4. A frog starts out at the central lilypad in an infinite row of equally-spaced lilypads. With each hop, the frog travels from the lilypad he is at to either the lilypad directly to the right or the lilypad directly to the left (each a distance of 1 unit). After 6 hops, find the expected distance from the frog to his starting position.

   (A) $\frac{15}{4}$ (B) $\frac{9}{2}$ (C) $\frac{32}{9}$ (D) 3 (E) NOTA

5. One six-sided die is weighted such that the probability of getting a number n (from 1 to 6) is directly proportional to n. Dielan creates another six-sided die and claims it also has this property. Skeptical, you test if this is true by rolling the die 105 times and recording the results.

Outcome:	1	2	3	4	5	6
Count:	7	8	12	27	22	29

   Find the p-value of the corresponding χ^2-test comparing the observed counts of each value, and the expected counts. Round to three decimal places.

   (A) 0.426 (B) 0.618 (C) 0.631 (D) 0.461 (E) NOTA

6. If X is a random variable with $\mu_X = 5$, $\sigma_X = 4$, and Y is a random variable with $\mu_Y = 3$, $\sigma_Y = 3$, what is the expected value of $X^2 + Y^2 + X + Y$ if X and Y are independent?

   (A) 49
   (B) 67
   (C) 41
   (D) 50
   (E) NOTA

7. In a factory, the weights of (rather large) tennis balls are normally distributed with $\mu = 1$ lb, $\sigma = 0.5$ lbs. When packaged for distribution, 30 of these tennis balls are placed within a cardboard box, whose weight is also normally distributed with $\mu = 2$ lbs, $\sigma = 0.35$ lbs. When being shipped, 5 of these cardboard boxes full of tennis balls are placed within a delivery truck, whose weight is ALSO normally distributed with $\mu = 150$ lbs, $\sigma = 10.25$ lbs. What is the probability that the weight of the entire truck, filled with boxes, which are in turn filled with tennis balls, falls between 300 lbs and 350 lbs? Round to three decimal places.

(A) 0.315
(B) 0.250
(C) 0.731
(D) 0.254
(E) NOTA

8. Prabhas is making questions for another Rickards Invitational test. Each question he writes is either good or bad, with equal probability. Prabhas keeps writing questions until he has written 30 good questions. At that point, let Q be the total number of questions Prabhas has written. What type of distribution does Q follow?

(A) Binomial (B) Geometric (C) Negative Binomial (D) Hypergeometric (E) NOTA

9. A circle is drawn, and three points are picked on the circumference such that no two points are in the same position. If a triangle is drawn from these points, what is the probability that it contains the center of the circle?

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) NOTA

10. How many of the following are unbiased estimators of a normal population: mean, median, standard deviation, minimum, maximum, range?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

11. A game is played such that on a particular turn, a wheel that has two zones: a green zone (adds one point to total score) that has a 65% chance of getting landed on, and a red zone (subtracts one point from total score) that has a 35% chance of getting landed on, is spun. What is the probability that your score is 8 after 10 turns, if you start the game with 10 points? Assume that every turn is independent. Round to three decimal places.

(A) 0.071 (B) 0.059 (C) 0.073 (D) 0.083 (E) NOTA

12. Honty Mall invites you on a game show and presents you with 10 doors, one of which has a prize behind it while the other 9 have nothing. He lets you pick the door you think has a prize behind it, so you choose one at random. Then, he opens one of the remaining doors, showing that it has nothing behind it. With only 9 doors left, he asks if you would like to switch to one of the other 8 doors. If you decide to switch, you have a probability A of winning the prize. If you decide not to switch, you have a probability B of winning the prize. Find A/B.

(A) $\frac{8}{9}$ (B) $\frac{9}{8}$ (C) 1 (D) $\frac{10}{9}$ (E) NOTA

13. Prabhas and Mihir are arguing over the efficiency of a new invention that claims to create carrots from thin air. A random sample of 100 machines are tested to find out how many carrots they can make in a constant time of 1 hour. These machines create an average of 55 carrots per hour with a population standard deviation of 2.54 carrots. Prabhas uses this data to conduct a significance test against the null hypothesis that the mean production is 52 carrots per hour with an alternative hypothesis that the mean production is greater than 52 carrots per hour, using a 0.05 level of significance. Mihir suggests that the true mean production is actually 56 carrots per hour. Find the power of this test if Mihir given that Mihir is actually right.

(A) 0.991 (B) 0.987 (C) 0.961 (D) 0.836 (E) NOTA

Let's slow down a bit! Use the following information to answer the next 5 questions:

A youth baseball league is trying to find out exactly how good their total player population is from the number of home runs they had in the previous season. They randomly sample 20 players from their leagues; their home runs are listed below, alongside their ages:

Ages: $16, 12, 19, 13, 11, 17, 15, 13, 15, 16, 18, 14, 18, 13, 16, 13, 11, 14, 18, 17$
Home runs: $16, 2, 25, 7, 4, 20, 22, 0, 24, 17, 9, 19, 10, 23, 6, 3, 8, 21, 1, 18$

14. Plotting the homeruns on a histogram gives a distribution that matches which most closely to which characteristic?

(A) Normal (B) Symmetric (C) Bimodal (D) Left Skewed (E) NOTA

15. What is the slope of the linear regression line that correlates age and home runs plus the correlation coefficient of the relation? Round to three decimal places.

(A) 1.234 (B) 1.462 (C) 1.453 (D) -3.837 (E) NOTA

16. Another player's age and number of home runs are measured at (12, 10). Using the linear regression line calculated in the previous question, what is the residual for this particular player? Round to three decimal places.

(A) -0.587 (B) 0.587 (C) 19.413
(D) -19.413 (E) NOTA

17. Let's say the manager of this youth baseball league wants to make a confidence interval to approximate the mean number of homeruns of the entire league. She must use a t-distribution instead of a normal distribution for this sample because it does not pass which test of normalcy?

(A) Simpson's Paradox (B) 10% Rule (C) Central Limit Theorem
(D) Law of Large Numbers (E) NOTA

18. Let X be the sum of the upper and lower bounds of a 95% confidence interval for the mean number of homeruns in the entire league. Find $\lfloor X \rfloor$.

(A) 24 (B) 25 (C) 26 (D) 27 (E) NOTA

19. A new disease called MAOVID-19 can be found to exist only within 5% of MAO competitors. A test has been made for this particular disease, but it gives a false positive 10% of the time, while giving a false negative 5% of the time. What is the probability that the test will correctly determine whether someone has MAOVID-19? Round to three decimal places.

(A) 0.903 (B) 0.783 (C) 0.750 (D) 0.950 (E) NOTA

20. How many of the following are conditions that must be met for a probability distribution to be hypergeometric?

I. The value of the probability of success depends on previous trials.
II. Each trial is NOT independent.
III. The number of trials is variable.
IV. Each observation is either success or failure.

(A) 2 (B) 4 (C) 3 (D) 1 (E) NOTA

21. Ananya and Akhil both arrive at a MAO meeting at some time between 2:00 and 3:00. When Ananya arrives, she stays for 20 minutes (or until it is 3:00), then leaves. When Akhil arrives, he stays for 10 minutes (or until it is 3:00), then leaves. Find the probability that, at some point in the hour, both Ananya and Akhil are present in the meeting. Assume that Ananya and Akhil's arrival times are random and independent. Round to 3 decimal places.

(A) 0.315 (B) 0.431 (C) 0.516 (D) 0.569 (E) NOTA

22. X and Y are correlated normal random variables with a correlation coefficient of 0.28. Given that $\mu_X = 0$, $\sigma_X = 3$, $\mu_Y = 9$, $\sigma_Y = 3$, and $\mu_{X^4} = 97$, what is the probability that $X^2 + Y$ is between 15 and 25? Round to four decimal places.

(A) 0.5732 (B) 0.8212 (C) 0.4742 (D) 0.5959 (E) NOTA

23. If $P(A) = 0.56$ and $P(B) = 0.29$, find the sum of the minimum and maximum values of $P(A \cup B)$.

(A) 0.85 (B) 0.27 (C) 1.41
(D) 1.12 (E) NOTA

24. Mihir loves switching seats! On the first day of class he sits in Seat A, but every day he is in Seat A he has a 20% chance of switching to Seat B the next day. Meanwhile, every day he is in Seat B, he has a 10% chance of switching back to Seat A the next day. By the last day of class (180 days later), what is the probability that he is sitting in Seat B? Round to the nearest tenth.

(A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6 (E) NOTA

25. Scientists have come up with a brand-new drug that cures procrastination. They perform an experiment with two groups of 50 people: one group getting the experimental drug and one group getting a sugar-water mixture. They find that from the treatment group, 74% of the participants have been cured of their procrastination urges, while only 32% of the water group have been cured. The scientists decide to perform a two-sample z-test on the data to find out whether these results were significant. Since the two groups were from the same population, the scientists decide to use a pooled standard error. Find the value of this standard error. Round to four decimal places.

(A) 0.0928 (B) 0.0917 (C) 0.0998 (D) 0.1000 (E) NOTA

Let's take another break. Use the following information to solve the next 4 questions:

Researchers are trying to determine the effect that caffeine and sleep has on an individual's productivity. They believe that 9 hours of sleep, or 5 mg of caffeine will effectively increase a person's productivity throughout the day. To investigate this, researchers take a random sample of 50 young adults, and split them into groups of 2. One person takes a 5mg caffeine pill, while the other person takes a sugar pill. They then keep track of their productivity throughout the day. The researchers also decide to conduct a survey to support their beliefs about sleep. They randomly call houses and ask how long they sleep, and how productive they are throughout the day, but they decide to call in the middle of the night. They find out that the mean amount of time people sleep is 6.6 hours, with a standard deviation of 2.3 hours.

26. What type of experimental design is described most above?

(A) Block (B) Randomized (C) Stratified (D) Clustered (E) NOTA

27. The sugar pill matches most closely to which term in the context of experimental design?

(A) Level (B) Factor (C) Treatment (D) Placebo (E) NOTA

28. If the researchers are trying to test whether the general public are not getting enough sleep in the survey, what would an appropriate null and alternative hypothesis be?

(A) $H_0 = 6.6, H_a = 2$ (B) $H_0 = 9, H_a \neq 9$ (C) $H_0 = 9; H_a > 9$ (D) $H_0 = 9; H_a < 9$ (E) NOTA

29. There is an example of strong bias shown in the sampling method for the survey in the prompt above. What bias does this show?

(A) Undercoverage (B) Nonresponse (C) Wording (D) Voluntary Response (E) NOTA

30. Find the expected value of the pdf defined by:

$$p(x) = \begin{cases} 2^{-\lfloor x \rfloor} & x > 1 \\ 0 & otherwise \end{cases}$$

(A) 2 (B) 2.5 (C) 4 (D) 5 (E) NOTA